Calculus Warm Up #2- 4

Find the area of the region bounded by f(x) and the x-axis on [0, 1] using the limit definition.

$f(x) = x^3 + 2x$

HW Questions: p. 249

In Exercises 25–30, find the limit of $s(n)$ as $n \to \infty$.

27. $s(n) = \frac{81}{n^4}\left[\frac{n^2(n+1)^2}{4}\right]$

29. $s(n) = \frac{18}{n^2}\left[\frac{n(n+1)}{2}\right]$

In Exercises 31–36, use the properties of sigma notation to find a formula for the given sum of n terms. Then use the formula to find the limit as $n \to \infty$.

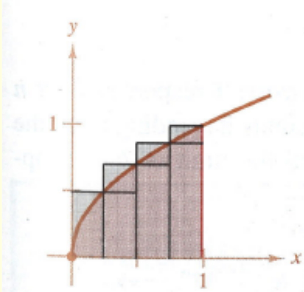
31. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{1}{n^3}(i-1)^2$

33. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \frac{16i}{n^2}$

35. $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left(1+\frac{2i}{n}\right)^3\left(\frac{2}{n}\right)$

In Exercises 37–42, use the upper and lower sums to approximate the area of the given region using the indicated number of (equal) subintervals.

37. $y = \sqrt{x}$

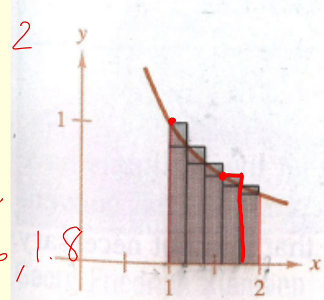


Lower Sum:
Rt endpts
1.2, 1.4, 1.6, 1.8, 2

upper Sum
left endpts
1, 1.2, 1.4, 1.6, 1.8

39. $y = \frac{1}{x}$

$n = 5$
$\Delta x = \frac{1}{5}$



$S(n) = 0.2\left[f(1.2) + f(1.4) + f(1.6) + f(1.8) + f(2)\right]$

$=$

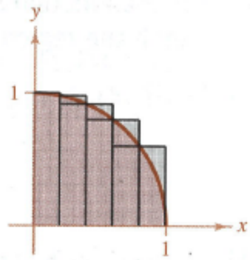
≈ 0.646

$S(n) = 0.2\left[f(1) + f(\right.$

$\approx$

In Exercises 37–42, use the upper and lower sums to approximate the area of the given region using the indicated number of (equal) subintervals.

41. $y = \sqrt{1 - x^2}$



Summary:

when given n = a particular number of subintervals

find $\Delta x = \dfrac{b-a}{n}$ on $[a, b]$

find Rt endpts $\Rightarrow$ $x_i = a + \Delta x \cdot i$

left endpts $\Rightarrow$ $x_i = a + \Delta x(i-1)$

approx area

$$A = \Delta x\left[f(x_1) + f(x_2) + \dots + f(x_n)\right]$$

Same as:

$$A = \sum_{i=1}^{n} \Delta x\left[f(x_i)\right]$$

Finding Actual Area

let $\Delta x \to 0$ approach   Same as let $n \to \infty$

$$A = \lim_{n \to \infty} \sum_{i=1}^{n} \Delta x \left[f(x_i) \right]$$

* Limit Definition of the Area under a Curve.

5.3 -Riemann Sums

-The Definite Integral

-Properties of The Definite Integral

Riemann "Rē - mahn"

Riemann is credited for generalizing the formula for use in accumulation applications beyond area under a curve.

Riemann Sums: $\sum_{i=1}^{n} \Delta x \left[f(x_i) \right]$

(Used to measure any accumulated change over an interval)

The Definite Integral: The limit of a Riemann Sum

Definition of the Definite Integral

For f(x) defined on [a, b]

$$\lim_{\|\Delta\| \to 0} \sum_{i=1}^{n} \Delta x\left[f(x_i)\right] = \int_a^b f(x)\,dx$$

$\|\Delta\|$ is called the norm of the partition

a = lower limit of integration
b = upper limit of integration

the largest width Δx when they are not uniform.

The Definite Integral

For f(x) continuous on [a, b] $\int_a^b f(x)\,dx$

Gives us a number.

The Indefinite Integral $\int f(x)\,dx$

Gives us a family of curves

(functions: F(x))

Evaluating a definite integral as a limit

$$\int_{-2}^{1} 2x \, dx \qquad f(x)$$

$$\lim_{n\to\infty} \sum_{i=1}^{n} \Delta x \left[f(x_i)\right]$$

$[-2, 1]$ $\quad \Delta x = \frac{1-(-2)}{n} = \frac{3}{n}$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{3}{n}\left[f\left(-2+\frac{3}{n}i\right)\right]$$

$$x_i = -2 + \frac{3}{n}i$$

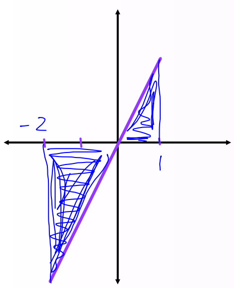
$$= \lim_{n\to\infty} \frac{3}{n}\sum\left(-4+\frac{6}{n}i\right)$$

$$= \lim_{n\to\infty} \frac{3}{\cancel{n}}\left(-4\cancel{n} + \frac{\cancel{6}3}{n}\cdot\frac{\cancel{n}(n+1)}{\cancel{2}}\right)$$

$$= \lim_{n\to\infty}\left(-12 + \frac{9n+9}{n}\right)$$

$$= \lim_{n\to\infty}\left(-12 + 9 + \frac{9}{n}\right)$$

$$= \boxed{-3}$$



THEOREM 5.7 THE DEFINITE INTEGRAL AS THE AREA OF A REGION

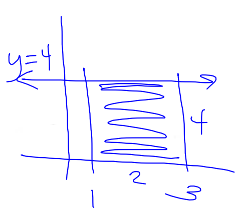
** p. 254

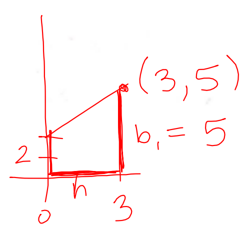
If f is continuous and nonnegative on the closed interval $[a, b]$, then the area of the region bounded by the graph of f, the x-axis, and the vertical lines $x = a$ and $x = b$ is given by

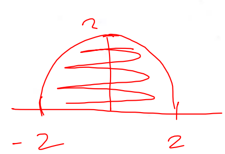
$$\text{area} = \int_a^b f(x)\,dx.$$

Sketch the region corresponding to each of the following definite integrals. Then evaluate each integral using a geometric formula.

(a) $\int_1^3 4\,dx = 8$ (b) $\int_0^3 (x+2)\,dx$ (c) $\int_{-2}^{2} \sqrt{4-x^2}\,dx$







$$A_{\text{Trap}} = \frac{(b_1+b_2)h}{2}$$

$$= \frac{(5+2)(3)}{2}$$

$$= \frac{21}{2}$$

$$A = \frac{1}{2}\pi(2)^2$$

$$= 2\pi$$

DEFINITION OF TWO SPECIAL DEFINITE INTEGRALS

1. If f is defined at $x = a$, then
$$\int_a^a f(x)\,dx = 0.$$
2. If f is integrable on $[a, b]$, then
$$\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx.$$

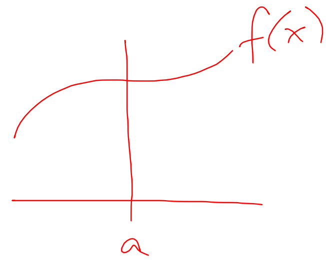
Evaluate the following definite integrals.

(a) $\int_2^2 \sqrt{x^2 + 1}\,dx$ (b) $\int_3^0 (x + 2)\,dx$

$= 0$

$= -\int_0^3 (x+2)\,dx$

$= -\frac{21}{2}$



THEOREM 5.8
ADDITIVE INTERVAL PROPERTY

If f is integrable on the three closed intervals determined by a, b, and c, then
$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx.$$

THEOREM 5.9
PROPERTIES OF DEFINITE INTEGRALS

If f and g are integrable on $[a, b]$ and k is a constant, then the following properties are true.

1. $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$
2. $\int_a^b [f(x) \pm g(x)]\,dx = \int_a^b f(x)\,dx \pm \int_a^b g(x)\,dx$

Evaluate

$$\int_1^3 (-x^2 + 4x - 3)\,dx$$

using the following values:

$$\int_1^3 x^2\,dx = \frac{26}{3}, \qquad \int_1^3 x\,dx = 4, \qquad \text{and} \qquad \int_1^3 1\,dx = 2.$$

$-\int_1^3 x^2\,dx + 4\int_1^3 x\,dx - 3\int_1^3 dx$

$-\frac{26}{3} + 4(4) - 3(2)$

HW:

p. 258, # 1 - 29 odd

* Use reasoning on #27.